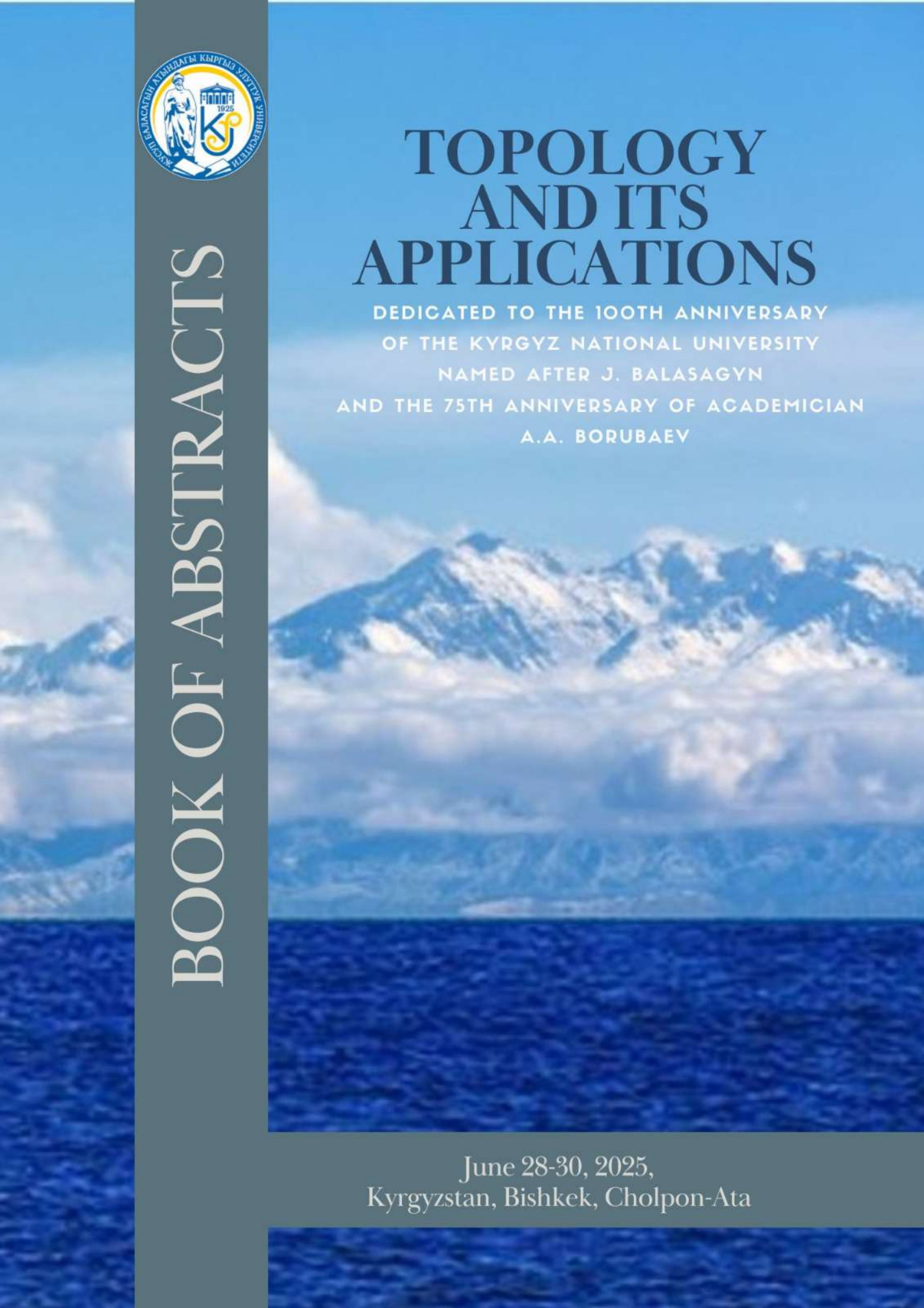

BOOK OF ABSTRACTS

TOPOLOGY AND ITS APPLICATIONS

DEDICATED TO THE 100TH ANNIVERSARY
OF THE KYRGYZ NATIONAL UNIVERSITY
NAMED AFTER J. BALASAGYN
AND THE 75TH ANNIVERSARY OF ACADEMICIAN
A.A. BORUBAEV

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KYRGYZ NATIONAL UNIVERSITY NAMED AFTER J. BALASAGYN

Institute of Mathematics and Informatics of Kyrgyz National University named after J. Balasagyn

Department of Algebra, Geometry, Topology and Teaching of Higher Mathematics named after Academician A.A. Borubaev

BOOK OF ABSTRACTS

International Scientific Conference “Topology and its Applications” dedicated to the 100th Anniversary of the Kyrgyz National University named after J. Balasagyn and the 75th Anniversary of Academician A.A. Borubaev

Bishkek 2025

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Contents

Generalization of the main principles of functional analysis

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These principles are generalized to the class of multi-normal spaces, which makes it possible to introduce multinorms in any locally convex linear topological space.

Definition 1. A mapping $\|\cdot\|_\tau\colon L^+ \to \mathbb{R}_\tau$ of a linear space L (over the real number field) into a space $\mathbb{R}_\tau^+$ is called a $\tau-$ norm or multinorm (if τ is not fixed) on the linear space L and a pair $(L, \|\cdot\|_\tau)$ is $\tau-$ normalized or multinormalized space if the following axioms are satisfied:

1. $\|x\|_\tau = \theta$ if and only if and only if x is a zero element of linear space L and θ is a point of space $\mathbb{R}_\tau^+$ all coordinates of which are zero;

2. $\|\lambda \cdot x\|_\tau = |\lambda| \cdot \|x\|_\tau$ for any scalar $\lambda \in R$ and any $x \in L$;

3. $\|x + y\|_\tau \leq \|x\|_\tau + \|y\|_\tau$ for all $x, y, z \in L$.

If $(L\|\cdot\|_\tau)$ is a multinormalized space and $\rho_\tau(x, y) = \|x - y\|_\tau$ then we obtain a $\tau-$metric space, which in turn gives rise to a linear normalized space.

A complete $\tau-$normalized space $(L\|\cdot\|_\tau)$ is called a $\tau-$Banach or amultiBanach space.

Theorem 1. Any $\tau-$Banach spaces and only these are limits of projective spectra of length τ composed of Banach spaces.

Theorem 2. (about the open mapping. Let $f\colon (E, \|\cdot\|_\tau) \to (L, \|\cdot\|_\tau)$ be a linear continuous mapping the $\tau-$Banach space $(E\|\cdot\|_\tau)$ to the $\tau-$Banach space $(L\|\cdot\|_\tau)$. Then the mapping f is open.

Theorem 3. (about the inverse operator). Let $A\colon (E, \|\cdot\|_\tau) \to (L, \|\cdot\|_\tau)$ be a linear continuous operator, mutually identically mapping $\tau-$Banach space $(E\|\cdot\|_\tau)$ to $\tau-$Banach space $(L\|\cdot\|_\tau)$. Then the inverse operator A^{-1} is continuous.

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Conformal Fedosov structures in symmetric and recurrent (pseudo-)Riemannian spaces

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This abstract is devoted to the study of the fundamental equations of conformally Fedosov structures in symmetric and recurrent (pseudo-) Riemannian spaces. Based on [1], we introduce the following definition:

Definition 1. A *conformally Fedosov manifold* $A_n = (M, [J], [\nabla])$ is a conformally symplectic manifold $(M, [J])$ in which, for the skew-symmetric form J, the equation $\nabla_k J_{ij} = J_{i[j a_{k]}}$ holds, where a_i are components of a linear form.

In [1] it is shown that these formulas are equivalent to the following equations:

$$\nabla_k J^{ij} = a^{[i \delta^{J]}_k},$$

where J^{ij} are the components of the matrix inverse to $\|J_{ij}\|$, i.e., $J_{ac}J^{bc} = \delta^a_b$, and δ^a_b denote the Kronecker delta. The tensors J_{ij} and J^{ij} determine the Fedosov manifolds; we call them a *Fedosov structure.*

In [2,3], a fundamental system of partial differential equations of Fedosov structure was found in Cauchy form.

We recall that a space is called symmetric or recurrent if its Riemann tensor R is absolutely parallel or recurrent, respectively, i.e.:

$$\nabla R = 0 \quad \text{or} \quad \nabla R = \varphi \diamond R$$

We have proven the following:

Theorem 1. Conformally Feodosov structures cannot exist in symmetric and recurrent (pseudo-) Riemannian spaces $V_n \quad (n \geq 4)$ of non-constant curvature.

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On bitopological weak R-separability

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Let X be a topological space. A family $\mathcal{B}$ of open subsets of X is said to be a base of X if for each $x \in X$ and for every its open neighborhood Ox there exists a $U \in \mathcal{B}$ such that $x \in U$ and $U \subset Ox$. The cardinal

$$w(X) = \aleph_0 + \min\{|\mathcal{B}| : \mathcal{B} \text{ is a base of } X\}$$

is called the *weight* of X. Here and further during in the paper symbol $\aleph_0$ means the cardinality of an infinite countable set.

A family $\mathcal{C}$ of subsets of X is said [3] to be a π-*net* of X if for each $x \in X$ and for every its open neighborhood Ox there exists a nonempty $U \in \mathcal{C}$ such that $U \subset Ox$. A π-*net* $\mathcal{C}$ consisting of open subsets of X is said [3] to be a π-*base* of X. The cardinal

$$\pi w(X) = \aleph_0 + \min\{|\mathcal{C}| : \mathcal{C} \text{ is a } \pi\text{-base of } X\}$$

is the π-*weight* of X.

It is clear that any base is a π-base of a given space. But the converse is not true. So, we have $\pi w(X) \le w(X)$. There exist examples of strict inequality. For example, we have $\pi w(L) = \aleph_0 < c = w(L)$ for the Niemytzki plane L.

Recall, a subset A of a topological space X is said [2] to be *dense* in X if $A \cap U \neq \varnothing$ for every open set U in X. It is easy to see that a subset A of a topological space X is dense in X if and only if $A \cap U \neq \varnothing$ for every $U \in B$ for some base $\mathcal{B}$ of X.

We claim that the requirement in the definition of a dense set may be weakened.

Proposition 1. A subset A of a topological space X is dense in X if and only if $A \cap U \neq \varnothing$ for every $U \in \mathcal{C}$ for some π-base $\mathcal{C}$ of X.

The *density* $d(X)$ of X is the smallest cardinality $\max\{|A|, \aleph_0\}$ of dense subsets in X, i.e.

$$d(X) = \aleph_0 + \min\{|A| : A \text{ is a dense set in } X\}.$$

It is well-known that $d(X) \le w(X)$. Noted above Proposition 1 yields the following remarkable statement which improves the last inequality.

Corollary 1. The density of any topological space X is not greater than its π-weight, i.e., $d(X) \le \pi w(X)$.

The inequality in Corollary generates the following question in a natural way.

Question 1. Is there a topological space X with $d(X) \le \pi w(X)$?

Now we will consider weak density of topological spaces. At first, we recall the concept of π-base by the sense in [2]. Let $\mathcal{C}$ be a family of subsets of a space X. If every open nonempty set contains some element of $\mathcal{L}$ as a nonempty subset, then the family $\mathcal{C}$ is said to be *dense* in X. A dense in X family $\mathcal{C}$ of open sets is called as a π-*base* of X.

Let $\mathcal{U}$ denote sets. For a topological space X a *weak density* $wd(X)$ is the smallest cardinal number $\{\tau = \max\{|\mathcal{U}|, \aleph_0\}$ such that there exists a π-base $\mathcal{C}$ admitting a representation as a union $\bigcup\limits_{\alpha \in \mathcal{U}} \mathcal{C}_\alpha$ of τ centered systems $\mathcal{C}_\alpha$ of open subsets in X for each $\alpha \in \mathcal{U}$. In the other words,

$$wd(X) = \aleph_0 + \min\left\{ |\mathcal{U}| : \begin{array}{l} \text{for each } \alpha \in \mathcal{U} \text{ there exists a centered system } \mathcal{C}_\alpha \\ \text{of open in } X \text{ sets such that } \bigcup\limits_{\alpha \in \mathcal{U}} \mathcal{C}_\alpha \text{ is a } \pi\text{-base of } X \end{array} \right\}.$$

A space X with $wd(X) = \aleph_0$ is said to be a *weak separable* space.

Remark 1. Weak density of a topological space X is τ if and only if there exists a π-net of subsets of X admitting a union of τ centered system of sets.

It is well known that $wd(X) \le d(X)$ for every space $(X, \mathcal{O})$. Indeed, let A be a dense subset of X with $d(X) = |A|$. For a point $x \in A$ a system

$$\mathcal{B}_x = \{U \in \mathcal{O} : x \in U\}$$

is a centered system of open in X sets. Since each $\mathcal{B}_x$ is a neighborhood system, the union $\mathcal{B} = \bigcup\limits_{\alpha \in A} \mathcal{B}_\alpha$ is a base of the topology $\mathcal{O}$, consequently, $\mathcal{B}$ is a π-base in X. Then

$$wd(X) \le |A| = d(X)$$

With respect to dense sets in a topological space, the question arises about the definition of the concept of 'weak dense' sets. In connect with this question we consider the following example.

Example 1. Consider the topological space $X = [0, \omega_1)$ - the set of all countable ordinals endowed with the topology $\mathcal{O}$ consisting of all sets with compact complements and the empty set:

$$\mathcal{O} = \{U \subset X : X\backslash U \text{ is a compact set}\} \cup \varnothing$$

The system $\mathcal{B} = \{U \subset X : X \setminus U \text{ is a compact set}\}$ is both a base and a π-base of $(X, \mathcal{O})$. Note that $w(X) = \pi(X) = d(X) = \aleph_1$; here $\aleph_1$ is the first uncountable cardinal number.

The system $\mathcal{B}$ can be considered as a union of a unique centered system $\mathcal{B}$ of open sets. To explain more detailly, consider the unique compact extension $Y = [0, w_1] = X \cup w_1$ of X. Then

$$Cl(\mathcal{B}) = \{U \cup (w_1) : U \subset X \text{ and } X\backslash U \text{ is a compact set}\}$$

is a centered system of compact sets in Y. Clearly, $\cap Cl_Y(\mathcal{B}) = \{w_1\}$. That is why $\mathcal{U} = \{w_1\}$ and $\bigcup\limits_{\alpha\in\mathcal{U}} \mathcal{B}_\alpha = \mathcal{B}$, consequently, $|U| = 1$ and $wd(X) = \aleph_0$.

It is worth marking that $|U| = \{w_1\}$ is a dense subset of Y. Consequently, $d(Y) = \aleph_0$. We obtain $d(X) = \aleph_1 > \aleph_0 = d(Y)$ despite of $X \subset Y$. Next, we have

$$\mathcal{U} \cap X \neq \varnothing \ (1)$$

It means, elements forming a centered system may not belong to the space, but belong to some its extension. Kind of such objects we will call as *spatial* points.

Remind, every element of a dense set is a point of the space; such an element we will call as an *inner* point. Thus, unlike dense sets a weak dense set may determine using spatial points. In other words, the set $\mathcal{U}$ of indices in the definition of weak density of a topological space X need not lie in X.

In Example 1 an element w_1 forms a unique centered system $Cl_Y(\mathcal{B}) = \{[\alpha, w_1] : \alpha < w_1\}$ but $w_1 \notin X$ and $w_1 \in Y$, i.e. $\mathcal{U} = \{w_1\} \subset Y\backslash X$.

The raised phenomenon (1) does not allow us to research weak dense versions of collections of all dense subsets studied in [1]. Therefore, according Remark 1 we will work with π-nets of spaces.

Let $\mathcal{A}$ and $\mathcal{B}$ be sets consisting of families of subsets of an infinite set X. Then: $S_1(\mathcal{A}, \mathcal{B})$ is the selection hypothesis: for each sequence $\{A_n : n \in \mathbb{N}\}$ of elements of $\mathcal{A}$ there is a sequence $\{b_n : n \in \mathbb{N}\}$ such that for each $n, b_n \in A_n$, and $\{b_n : n \in \mathbb{N}\}$ is an element of $\mathcal{B}$.

Throughout this note the entry (X, τ_1, τ_2), sometimes written simply X, will be a bitopological space (shortly bispace), i.e. the set X endowed with two topologies τ_1 and τ_2. For a subset A of X, $Cl_i(A)$ will denote the closure of A in $(X, \tau_i), i = 1, 2$.

Definition 2. A system $\mathcal{C}$ of sets in X is a bi-π-net in X if $\mathcal{C}$ is a π-net in both (X, τ_1) and (X, τ_2). X is weak *biseparable* (shortly, wd-separable) if there is a π-net admitting countable union of centered systems of sets in both (X, τ_1) and (X, τ_2).

A pairwise π-base of X is [1] a collection of pairs $(P_\alpha, Q_\alpha) \in \tau_1 \times \tau_2, \alpha \in \Lambda$, such that for every $(U, V) \in \tau_1 \times \tau_2$ there is $\alpha \in \Lambda$ such that $P_\alpha \subset V$ and $Q_\alpha \subset U$.

Denote by $\mathcal{C}_1$ and $\mathcal{C}_2$ the collections of all π-nets of (X, τ_1) and (X, τ_2) admitting countable union of centered systems, respectively.

Definition 3. We say that X is weak $R_{(\tau_i,\tau_j)}$-separable if $S_1(\mathcal{C}_i, C_j)$ holds.

Fact 1. Weak $R_{(\tau_i,\tau_j)}$-separability implies weak separability of (X, τ_j).

Fact 2. If $\tau_1 \leq \tau_2$, then

2.1. Weak $R_{(\tau_1,\tau_2)}$-separability implies (X, τ_1) is weak R-separable (and, by Fact 1, (X, τ_2) is separable).

2.2. (X, τ_1) is weak R-separable $\Rightarrow$ (X, τ_1, τ_2) is weak $R_{(\tau_1,\tau_2)}$-separable $\Rightarrow$ (X, τ_1) is weak separable.

Fact 3. If Y is either wd-dense or wd-open in (X, τ_1, τ_2) and X is weak $R(\tau_i, \tau_j)$-separable, then Y is also weak $R_{(\tau_i,\tau_j)}$-separable.

Theorem 1. If X has a countable pairwise π-net, then X is weak $R_{(\tau_i,\tau_j)}$-separable, $i, j = 1, 2$.

With respect to results in [1] the following questions of independent interest arise:

Question 2. Let (X, τ_1, τ_2) be a wd-separable bitopological space with countable weak (τ_i, τ_j)-strong fan tightness. Is X $R_{(\tau_i,\tau_j)}$-separable?

Question 3. Let (X, τ_1, τ_2) be a wd-separable bitopological space with countable weak (τ_i, τ_j)-strong fan tightness. Is X $M_{(\tau_i,\tau_j)}$-separable?

The missing information can be found in [1].

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The space of G-permutation degree and uniform spaces

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We study the behavior of various compact-type properties (pre-compactness, local compactness, uniform linkedness) of uniform spaces under influence of the functor SP^n_G of G-permutation degree. It is proved that this functor preserves pre-compact-ness local compactness, uniform linkdness and star-finiteness of uniform spaces. Also, the functor SP^n_G preserves (perfect) uniformly continuous mappings.

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Generalized universally measurable spaces

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V. Fedorchuk, A. Chisogidze, and T.Banach in 2003 and V. Bogachev in 2024 posed the following questions: (i) is it true that $P_\tau(X)$ is C-embedded in $P_\sigma(X)$? (ii) is it true that $P_R(X)$ is C-embedded in $P_R(\beta X)$ if and only if X is pseudocompact, where P_σ, P_τ, and P_R are probability σ-additive functors on the Baer σ-algebra, τ-additive and Radon measures on the space X? The answers to these questions are negative. But if instead of probability measures we consider the corresponding alternating measures M_σ, M_τ and M_R, then the situation changes. It is proved that (i) $M_\tau(X)$ is C-embedded in $M_\sigma(X)$; (ii) $M_R(X)$ is C-embedded in $M_R(\beta X)$ if and only if X is pseudocompact.

The question of C-embedding of measure spaces is an extension of the question about the coincidence of measure spaces, which is a development of the classical concepts of universally measurable and universally measured zero sets. A general theorem is obtained, the consequence of which are the results we have stated.

For Tichonoff space X, the most important measure spaces on X are identified with subsets of the space $M(\beta X)$ of all regular Radon measures on the Stone-Chech compactification βX, which makes it possible to topologize these spaces with a star-weak topology: $M_f(X) = M(\beta X)$ are finite additive measures on the Baer σ-algebra $\mathcal{B}(X)$; $M_\sigma(X)$ are countably additive measures on $\mathcal{B}(X)$; $M_\tau(X)$ are τ-additive measures; $M_R(X) = M(X)$ are Radon measures; $M_\beta(X)$ are measures with compact support; $M_d(X)$ are discrete measures.

For $M \subset \mathbb{R}$, the classical concepts of universally measurable and universally measures zero sets of M are topological and are characterized as $M_\sigma(X) = M_R(X)$ and $M_\sigma(X) = M_d(X)$.

Let $a, b \in \{f, \sigma, \tau, R, \beta, d\}$.

The space X is called $[a, b]$-measurable if $M_a(X) = M_b(X)$; is a-measurable if $M_a(X) = M_R(X)$. The subfunctors $M_a^+(X) = M_a(X) \cap M^+(\beta X)$, $U_a(X) = M_a(X) \cap U(\beta X)$, $P_a(X) = M_a(X) \cap P(\beta X)$ non-negative measures, norm measures not exceeding unity, probability measures, respectively the $M_a(X)$ functor, are considered. For $Q \in \{M, M^+, U, P\}$ we call X $Q_{[a,b]}$-measurable if $Q_a(X)$ is C-embedded in $Q_b(X)$; we call X Q_a-measurable if $Q_R(X)$ is C-embedded in $Q_a(X)$.

Some problems of measure theory and topology are special cases of the following general question.

Problem 1. For $a, b \in \{f, \sigma, \tau, R, \beta, d\}$ and $Q \in \{M, M^+, U, P\}$, we describe $[a, b]$-measurable and $Q_{[a,b]}$-measurable spaces.

Theorem 1. For $a, b \in \{f, \sigma, \tau, R, \beta, d\}$, the space X is an $M_{[a,b]}$-measurable space if and only if any separable metrizable space Y, which is a continuous image X, is a $[a, b]$-measurable space.

There have been questions in the literature that can be formulated as follows: (i) is it true that any space is $P_{[\tau,\sigma]}$-measurable? (ii) is it true that the space X is $P_{[R,f]}$-measurable if and only if X is pseudocompact?

The answers to these questions are negative. But when the measure spaces change, the situation changes.

As a consequence of Theorem 1, we obtain: (i) any space X is $M_{[\tau,\sigma]}$-measurable (that is, $M_\tau(X)$ is C-embedded in $M_\sigma(X)$); (ii) the space X is $M_{[R,f]}$-measurable if and only if X is pseudocompact ($M_R(X)$ is C-embedded in $M(\beta X)$ if and only if X is pseudocompact).

Idempotent measures on Π-complete spaces

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A Tychonoff space X is said to be Π-complete if for every point $x \in \beta X \backslash X$ there exists a finite component cover ω of X which pricks out the point x in βX.

A set of all idempotent probability measures on X is denoted by $I(X)$. It is endowed with the topology τ_p of pointwise convergence. For $\mu \in I(X)$ sets

$$(\mu; \varphi_1, ..., \varphi_n : \theta) = \{v \in I(X) : |v(\varphi_i) - \mu(\varphi_i)| < \theta, i = 1, ..., n\}$$

form a base of $I(X)$ at μ. Here $\varphi_1, ..., \varphi_n \in C(X)$, and $\theta > 0$

For a positive integer n let $I_n(X)$ denote a set of all idempotent probability measures that support consists of no more than n points. Put $I_\omega(X) = \bigcup\limits_{n=1}^{\infty} I_n(X)$. An idempotent probability measure $\mu \in I_\omega(X)$ is said to be an idempotent probability measure with finite support.

For a compact Hausdorff space X, we put

$$I_f(X) = \left\{ \mu = \bigoplus_{i=1}^{n} \chi_\mu(x_i) \odot \delta_{x_i} \in I_\omega(X) \;\middle|\; \begin{array}{l} \text{there exists a point } x_{i_0} \in \operatorname{supp}\mu \text{ such that } \chi_\mu(x_{i_0}) = 0, \\ \text{and } \chi_\mu(x_i) \le -\frac{n}{n+1} \text{ for } i \ne i_0 \end{array} \right\}.$$

For a Tychonoff space X let βX be its Stone–Čech compactification (i.e. the maximal compact extension). Let $I_\beta(X) = \{\mu \in I(\beta X) : \operatorname{supp}\mu \subset X\}$. Elements $\mu \in I_\beta(X)$ are called as idempotent probability measures with compact support. It is well known [3] that $I_\beta(X)$ lies in the compact Hausdorff space $I(\beta X)$ as an everywhere dense subset. Consequently, $I_f(\beta X)$ is some compact extension of $I_f(X) = I_f(\beta X) \bigcap I_f(X)$. The following result clarifies the phenomenon.

Theorem 1. For a Tychonoff space X the space $I_f(\beta X)$ is a perfect compactification of the space $I_f(X)$.

The following statement is the main result of our current note.

Theorem 2. For a Tychonoff space X, the space $I_f(X)$ is Π-complete if and only if X is Π-complete.

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On τ-base of topological spaces

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In recent researches an interest in the theory of cardinal invariants and their behavior under the influence of various covariant functors is increasing fast. In [1], [2], [3], [4], [5], [6], [7], [8] the authors investigated several cardinal invariants under the influence of some weakly normal and normal functors and hyperspaces.

The concept of a τ-closed subset was introduced Juhasz I. in 1980 in his book [9]. In 1987, Arkhangel'skii A.V. introduced the classes of τ-continuous and strictly τ-continuous mappings and gave examples of their discrepancy with the class of continuous mappings [10]. In 2016, in the work [11] Okunev O. introduced the concept of τ-closure of a set and presented some criteria for the τ-continuity of mappings. In 2023, Georgiou D.N., Mamadaliev N.K., Zhuraev R.M. introduced the definitions of a τ-open set and a τ-interior operator. Using new concepts, they expanded O. Okunev's theorem and introduced new criteria for τ-continuity of mappings [12]. In the work [13] some properties of ω-bounded spaces were studied.

In this article, we proved some properties of families of τ-open sets, studied the properties of the space of τ-continuous mappings, as well as the properties of the e-density of topological spaces. Let X be a topological space and let A be a subset of X. We denote the closure of A in X by $\mathrm{cl}_X A$.

Throughout the paper all spaces are assumed to be topological spaces and τ be an infinite cardinal number.

Definition 1 [9]. Let X be a topological space. A set $F \subset X$ is called τ-closed in X if for each $B \subset F$ such that $|B| \leq \tau$, the closure of the set B in X lies in F.

It is known that every closed subset of a topological space is τ-closed. But the opposite is not always true.

Example 1 [12]. On the real line we will assume that all sets whose complement is countable are open, and we will also declare the empty set is open, i.e. the set of all real numbers $\mathbb{R}$ has the following topology:

$$\theta = \{\varnothing\} \cup \{U : U \subset \mathbb{R}, |\mathbb{R} \backslash U| \leq \omega\}.$$

Since every set whose cardinality of its complement does not exceed ω is open in this topological space, then an arbitrary countable set $B \subset \mathbb{R}$ is closed. Let's choose an arbitrary subset $M \subset \mathbb{R}$. Then every subset $B \subset M$, whose cardinality does not exceed ω, coincides with its closure, which means that $B \subset M$ implies that $cl_{\mathbb{R}} B \subset M$ for all $|B| \leq \omega$.

From the arbitrariness of the set M it follows that each subset of this space is ω-closed. In particular, the set of all irrational numbers in this space is ω-closed, but not closed.

I. Juhasz in his work [9] proved that the tightness of a topological space X does not exceed τ if and only if every τ-closed subset is closed.

Definition 2 [12]. Let X be a topological space. A set $U \subset X$ is called τ-open in X if its complement $X \backslash U$ is τ-closed.

Every subset of the space defined in Example 1 is τ-open.

Any τ-open set containing a point $x \in X$ is called a τ-neighborhood of this point.

The τ-closure of a subset A is defined as follows:

$$[A]_\tau = \bigcup \{\ cl_X B : \ \ B \subset A, \ \ |B| \leq \tau\ \}.$$

Recall that a subset A is τ-dense in X if $[A]_\tau = X$ [11].

For any subsets A and B of the space X the following relation holds: if $A \subset B$, then $[A]_\tau \subset [B]_\tau$.

Example 2. On the set of real numbers with the natural topology, we choose the set of all rational numbers. Let's find its ω-closure $[\mathbb{Q}]_\omega = \bigcup\{\ cl_{\mathbb{R}} B : \ \ B \subset \mathbb{Q}, \ \ |B| \leq \omega\}$. As a subset $B \subset \mathbb{Q}$, $|B| \leq \omega$ we take the set itself $\mathbb{Q}$, the closure of which coincides with the set of real numbers. This means $[\mathbb{Q}]_\omega = \mathbb{R}$, and we can conclude that the set of rational numbers on the Euclidean line is ω-dense.

Let Θ_τ be the family of all τ-open subsets in X. The family $\mathrm{B}_\tau \subset \Theta_\tau$ is called the τ-base of the topological T_1-space X, if every τ-open subset of $U_\tau \in \Theta_\tau$, $U_\tau \neq \varnothing$ can be represented as a union of some subfamily B_τ.

Theorem 1. *(An analogue of the theorem of A.V. Arkhangelsky).* The family B_τ is a τ-base of the topological space X if and only if for every element x from X and for every τ-neighborhood $V \in \Theta_\tau$ of x there exists $U \in \mathrm{B}_\tau$ such that $x \in U \subset V$.

Remark 1. The base of a topological space is not always a τ-base.

Example 3. Consider the topological space given in Example 1. Since the complement of every set whose cardinality does not exceed ω is open in this topological space, then an arbitrary set $B \subset \mathbb{R}$ for which $|B| \leq \omega$ is closed. Let us choose an arbitrary subset $M \subset \mathbb{R}$. Then every subset $B \subset M$ whose cardinality does not exceed ω coincides with its closure, which means that from $B \subset M$ it follows that $\mathrm{cl}_{\mathbb{R}} B \subset M$ for all $|B| \leq \omega$. From the arbitrariness of the choice of M it follows that every subset of this space is ω-closed, and hence every subset is ω-open. As the base of this space, we can choose a family of subsets of the form $\mathrm{B} = \{U : U \subset \mathbb{R}, |\mathbb{R}\backslash U| \leq \omega\}$. Since each element of B is an infinite set, it is impossible to represent the ω-open subset $\{x\}$, where $x \in \mathbb{R}$, as a union of some subfamily of the family B. Therefore, B is not an ω-base.

Remark 2. A τ-base is not always a base.

Example 4. The ω-base $\mathrm{B}_\omega = \{\{x\} : x \in \mathbb{R}\}$ is not a base of the space $(\mathbb{R}, \theta)$ from Example 2 because its elements are not open sets.

Definition 3. The family $\mathrm{B}_\tau(x)$ of τ-neighborhoods of a point x is called a τ-base of the topological space X at point x if for every τ-neighborhood V of point x there exists $U \in \mathrm{B}_\tau(x)$ such that $x \in U \subset V$.

Definition 4. Let X be a topological T_1-space and for every x from X a τ-base $\mathrm{B}_\tau(x)$ of the space X. The family $\{\, \mathrm{B}_\tau(x) : \ x \in X \,\}$ is called a system of τ-neighborhoods of the topological space X.

Theorem 2. Any system of τ-neighborhoods $\{\, \mathrm{B}_\tau(x) : \ x \in X \,\}$ has the following properties:

($\mathrm{B}_\tau P1$) For every element x of X we have that $\mathrm{B}_\tau(x)$ is non-empty and for every element U from $\mathrm{B}_\tau(x)$ we have that $x \in U$.

($\mathrm{B}_\tau P2$) If x belongs to $U \in \mathrm{B}_\tau(y)$ for some $y \in X$, then there is a set $V \in \mathrm{B}_\tau(x)$ such that $V \subset U$.

($\mathrm{B}_\tau P3$) For any elements U_1, U_2 of the family $\mathrm{B}_\tau(x)$ there is a set $V \in \mathrm{B}_\tau(x)$ such that $V \subset U_1 \cap U_2$.

Proposition 1. For any subset A of a topological space X, the following conditions are equivalent:

1) A point x belongs to $[A]_\tau$;

2) For every $\mathrm{B}_\tau(x)$ and every $U \in \mathrm{B}_\tau(x)$ we have $U \cap A \neq \varnothing$;

3) There is a system of neighborhoods $\mathrm{B}_\tau(x)$ such that $U \cap A \neq \varnothing$ for every $U \in \mathrm{B}_\tau(x)$.

Corollary 1. If U is a τ-open set and A is some subset of the space X disjoint with U, then $U \cap [A]_\tau = \varnothing$. In particular, if U and V are disjoint τ-open subsets, then $U \cap [V]_\tau = [U]_\tau \cap V = \varnothing$.

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Topological Properties of τ-Closed Subsets and Vietoris Topology

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Let X be a topological T_1-space and τ an infinite cardinal number. Denote by $\exp X$ the set of all closed subsets of the space X. A basis for the Vietoris topology defined on $\exp X$ is the family of sets of the form:

$$O\langle U_1, U_2, \ldots, U_n\rangle = \left\{ F \in \exp X \mid F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \neq \varnothing \text{ for } i = 1, \ldots, n \right\},$$

where $U_1, U_2, \ldots, U_n$ are open subsets of X [1].

Theorem 1. [1] Let X be a T_1-space and $X_0 \subset X$. Then the set

$$\{F \in \exp X \mid X_0 \subset F\}$$

is closed in $\exp X$.

Theorem 2. [1] If X is a T_1-space, then $\exp X$ is also a T_1-space.

Theorem 3. [1] If Y is a dense subset of X, then $\exp Y$ is a dense subset of $\exp X$.

In 2016, O. Okunev introduced the concept of a τ-closed subset.

Definition 1. [2] A subset $F \subset X$ is called τ-closed in the topological space X if for every subset $B \subset F$ with $|B| \leq \tau$, the closure of B in X lies within F.

In 2023, the authors of [3] introduced the notion of a τ-open subset and established connections between τ-open, τ-closed subsets, and τ-continuous mappings, which were introduced by A.V. Arkhangelskii in 1983 [4]
.

In 2023, [5] investigated some properties of τ-closed and τ-open subsets, as well as the properties of τ-closure, τ-interior, and τ-boundary. Examples were given to demonstrate similarities and differences between τ-closure, τ-interior, and τ-boundary and the usual closure, interior, and boundary of subsets.

In [6] , the theory of τ-closed subsets was extended, introducing certain cardinal invariants such as τ-density and the τ-Suslin's number. Examples were provided to illustrate the significance of these cardinal invariants.

In [7] , the concepts of τ-bases and τ-neighborhood systems in topological spaces were introduced, and their properties were studied. The space of τ-continuous mappings was constructed and shown to be a T_i-space whenever the image of τ-continuous mappings is a T_i-space for $i = 0, 1, 2, 3$.

Let $\exp^\tau X$ denote the family of all τ-closed subsets of X.

Let $U_1, \ldots, U_n$ be τ-open subsets of X.

Theorem 4. The family of sets

$$O\langle U_1, \ldots, U_n\rangle = \left\{ F \in \exp^\tau X \mid F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \neq \varnothing \text{ for all } i = 1, \ldots, n \right\}$$

forms a basis for some topology on $\exp^\tau X$.

Theorem 5. If X is regular, then $\exp^\tau X$ is Hausdorff.

Theorem 6. Let X be a T_1-space and $X_0 \subset X$. Then the set

$$\{F \in \tau\text{-}\exp X \mid X_0 \subset F\}$$

is closed in the space $\tau\text{-}\exp X$.

Proposition 1. The space $\exp(X)$ is a dense subspace of $\exp^\tau(X)$.

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Pseudo-open maps and their behavior under symmetric power constructions

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In this paper, we illustrate that the symmetric power functor preserves almost-open mappings between compact Hausdorff spaces, demonstrating the stability of this topological property under finite quotient constructions.

On the n^{th} power X^n of the compact X, the permutation group S^n acts as the group of coordinate permutations. The set of orbits of this action with the quotient topology is denoted by SP^nX. Consider the quotient mapping

$$\pi_n^s : X^n \to SP^nX$$

that associates to each point $x = (x_1, x_2, \ldots, x_n) \in X^n$ the orbit of this point. Thus, the points of the space SP^nX are finite subsets (equivalence classes) of the product X^n.

In this setting, two points $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ are considered equivalent if there exists a permutation $\sigma \in S^n$ such that $y_i = x_{\sigma(i)}$ for all $i = 1, 2, \ldots, n$ [1].

Let $f : X \to Y$ be a continuous mapping between compact Hausdorf spaces X and Y. For the equivalence class $[(x_1, x_2, \ldots, x_n)] \in SP^nX$, put

$$(SP^nf)([(x_1, x_2, \ldots, x_n)]) = [(f(x_1), f(x_2), \ldots, f(x_n))].$$

This defines a mapping

$$SP^nf : SP^nX \to SP^nY.$$

It is easy to verify that the operation SP^n constructed in this way is a covariant functor in the category Comp of compact spaces and their continuous mappings.

Definition 1. A mapping f is called pseudo-open if for each $y \in Y$ and each neighborhood U of $f^{-1}(y)$ in X, $f(U)$ is a neighborhood of y in Y [2, 3].

Theorem 1. Let $f : X \to Y$ be a pseudo-open mapping. Then the induced map

$$SP^nf : SP^nX \to SP^nY$$

is also pseudo-open for every $n \in \mathbb{N}$.

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On Tightness-Type Properties of the Space of Weakly Additive Functionals

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This paper investigates cardinal invariants of the space of weakly additive functionals with finite supports. In particular, we study tightness-type properties such as tightness $t(X)$, minimal tightness, functional tightness $t_0(X)$, local density $ld(X)$, and the Hewitt number $q(X)$ (for basic definitions see [1] and [2]).

Special attention is given to the functor O_n, which assigns to each compact space X the space $O_n(X)$ consisting of weakly additive, order-preserving, normed functionals with support of size at most n. It is shown that the functor O_n preserves a number of important topological properties of the original space. In particular, for any infinite compact space X, the following equalities hold:

$$t(O_n(X)) = t(X), ld(O_n(X)) = ld(X), q(O_n(X)) = q(X)$$

Moreover, the functor O_n is extended to the category $Tych(\tau)$ of Tychonoff spaces and strictly τ-continuous maps. It is proven that the induced map $O(f)$ is strictly τ-continuous for any strictly τ-continuous function $f : X \to Y$. The obtained results generalize several known theorems on the behavior of car-dinal invariants under normal and weakly normal functors.

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On $\tau-$placedness of the space of compactmaximal linked systems

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A system $\xi = \{F_\alpha : \ \alpha \in A\}$ of closed subsets of a space X is called linked, if any two elements of ξ intersect. Each linked system can be filled up to a maximal linked system (MLS), but such a completion is not unique [2]. The set of all maximal linked systems of a space X we denoteby λX.

For a closed set $A \subset X$ we suppose $A^+ = \{\xi \in \lambda X : \ A \in \xi\}$. The family of sets in the form A^+ becomes a closed subbase in the space λX. For an open set $U \subset X$ we get

$$O\left(U\right) = \left\{\xi \in \lambda X : \text{there exists such } F \in \xi \text{ that } F \subset U\right\}.$$

The family of all the sets of the form $O\left(U\right)$ covers the set $\lambda X \ \left(O\left(X\right) = \lambda X\right)$, so it becomes a subbase of a topology on X. The set λX with this topology, is called the superextension of X. The family of all sets in the

$$O\left(U_1,\ U_2,\ \ldots,\ U_n\right) = \left\{\xi \in \lambda X : \text{ for } i - 1,\ 2,\ \ldots,\ n \text{ there exists such } F_i \in \xi \text{ that } F_i \subset U_i\right\}.$$

A maximal linked system ξ of a space X is called a *compact maximal linked system* if ξ contains at least one compact element [3].

We denote by $\lambda_c X$ the set of all compact maximal linked systems of the space X [3].

We say that $A \subset X$ is a set *of type* G_τ in X if there is a family γ of open sets in X such that $A = \cap\gamma$ and $|\gamma| \leq \tau$. A set $A \subset X$ is called $\tau-$*placed* in X if for each point $x \in X \backslash A$ there is a set P of type G_τ in X such that $x \in P \subset X \backslash A$ [1].

Put $q\left(X\right) = \min\left\{\tau \geq \omega : \ X \text{ is } \tau- \text{ placed in} \beta X\right\}$; $q\left(X\right)$ is called the *Hewitt-Nachbin number* of X. We say that X is a $Q_\tau-$*space* if $q\left(X\right) \leq \tau$.

This allows us to give the following definition: put $m\left(X\right) = \min\left\{\tau \geq \omega : \ X \text{ is an } m_\tau - \text{space}\right\}$[1].

Theorem 1. If $q\left(\lambda_c X\right) \leq \tau$ and $m\left(\lambda X\right) \leq \tau$, then $\lambda_c X$ is $\tau-$placed in λX.

Corollary 1. If $m\left(\lambda X\right) \leq \tau = d\left(X\right)$, then $\lambda_c X$ is $\tau-$placed in λX.

Corollary 2. If λX is Moscow space and X is separable, then $\lambda_c X$ is $\omega-$placed in λX.

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Inverse systems of intuitionistic fuzzy topological spaces of soft sets

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Inverse systems of intuitionistic fuzzy topological spaces of soft sets are defined, and it is proven that they form a category. The existence and uniqueness of the inverse limit in this category are established. It is also demonstrated that the inverse limit is a functor, and the properties of the inverse limit space are examined. Furthermore, the relationship between inverse systems of intuitionistic fuzzy topological spaces of soft sets and inverse systems of soft bitopological spaces is investigated. Finally, by introducing the quotient space and topological sum operations for intuitionistic fuzzy topological spaces of soft sets, direct systems in this category are explored, along with the existence of their limits.

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Different topologies on the space of G-symmetric power

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Let G be any subgroup of the group S_n. Then it also acts on X^n as permutation group of coordinates. Consequently, it generates a G-symmetric equivalence relation on X^n. The quotient space of the product X^n under the G-symmetric equivalence relation, is called G-symmetric power of the space X and is denoted by $SP^n_G X$. The operation SP^n_G is also the covariant functor in the category of compacts and is said to be a functor of G-permutation degree. If $G = S_n$ then $SP^n_G = SP^n$. If the group G consists only of unique element then $SP^n_G = \prod^n$. Moreover, if $G_1 \subset G_2$ for subgroups G_1, G_2 of the permutation group S_n then we get a sequence of the factorization of functors [2]:

$$\prod^n \to SP^n_{G_1} \to SP^n_{G_2} \to SP^n$$

Let $f : X \to Y$ be a continuous mapping. For a class equivalence $[(x_1, x_2, ..., x_n)] \in SP^n X$ put

$$SP^n f[(x_1, x_2, ..., x_n)] = [(f(x_1), f(x_2), ..., f(x_n))]:$$

Thereby, a mapping is defined $SP^n_G f : SP^n_G X \to SP^n_G Y$. It is easy to check that the operation SP^n_G so constructed is a normal functor in the category of compacts. This functor is called the functor of G-permutation degree.

Let τ be an infinite cardinal number and let X be a topological space. A subset F of X is called τ-closed in X [4] if for each $B \subset F$ such that $|B| \leq \tau$, the closure of the set B in X lies in F.

It is known that every closed subset of a topological space is τ-closed. But the opposite is not always true (see [3]).

The τ-closure of a subset A is defined as follows:

$$[A]_\tau = \bigcup \{\mathrm{cl}_X B : \ B \subset A, \ |B| \leq \tau\}.$$

Recall that a subset A is τ-dense in X if $[A]_\tau = X[1]$.

Lemma 1. Let X and Y be topological spaces and let A and B be τ-open subsets in X and Y respectively. Then Cartesian product $A \times B$ is τ-open in $X \times Y$.

We are going to introduce two topologies different from Tychonov topology on X^n. Let (X, θ) be a topological space. The product space with the Tychonov topology we denote by (X^n, θ^n).

Proposition 1. The family

$$\gamma_1 = \{U_1 \times U_2 \times ... \times U_n \subset X^n : \quad U_i \text{ is } \tau - \text{open in } (X, \theta) \text{ for each } i = 1, 2, ..., n\}$$

of subsets of X^n satisfies conditions B1)-B2) of base, so the family

$$\theta^n_\tau = \{\cup\gamma : \quad \gamma \subset \gamma_1\}$$

is a topology in X^n.

It easy to check that the family

$$\gamma_2 = \{P \subset X^n : \quad P \text{ is } \tau - \text{open in } (X^n, \theta^n)\}$$

of subsets of X^n satisfies conditions B1)-B2) of base, so the family

$$(\theta^n)_\tau = \{\cup\gamma : \quad \gamma \subset \gamma_2\}$$

is a topology in X^n.

Finally just we get two distinct topological spaces (X^n, θ^n_τ) and $(X^n, (\theta^n)_\tau)$.

Proposition 2. The topology θ^n_τ is weakly than the $(\theta^n)_\tau$.

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On some compactifications locally compact spaces and absolute retracts

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The topological properties of the Alexandrov and Freudenthal compactifications of locally compact spaces are considered. It is shown that the compactifications αX are covariant functors in the category $LComp$–locally compact spaces and continuous proper mappings into themselves. The topological properties of the Alexandrov, Freudenthal extension and its growth for any connected, locally connected locally compact space are studied. Below we show in what cases the Freudenthal compactification εX and the Alexandrov extension αX of a locally compact space X are a Hilbert manifold, and the Freudenthal compactification εX and the compactification αX of some subspaces of the space–$P(X)$ of all probability measures that are a Q–(Hilbert)–manifold.

Definition 1. A closed set A of a space X is called a Z-set in X if the identity mapping id_X of X can be arbitrarily closely approximated by mappings $f : X \to X \setminus A$

A set $A \subset X$ is called homotopy dense in X if there exists a homotopy $h(x,t) : X \times [0,1] \to X$ such that $h(x,0) = \mathrm{id}_X$ and $h(X \times (0,1]) \subset A$ [2].

A non-empty subset $A \subset X$ is homotopy negligible (or vague, i.e. nowhere dense) in X if $X \setminus A$ is homotopy dense in X.

For a non-compact space X, εX and αX denote the Freudenthal and Alexandrov extensions, respectively.

Theorem 1. For any locally compact ANR space X and its compactification εX has place:

a) X is homotopy dense in εX;

b) the embedding $X \hookrightarrow \alpha(X)$ is a fine homotopy equivalence;

c) the set $C(Q,X)$ is everywhere dense in $C(X,\varepsilon X)$;

Corollary 1. For any open subset $U \subset R$ of a line R, the compactification εU contains U as a homotopy dense set. Therefore, the line R is homotopy dense in εR.

Corollary 2. For any local compact ANR space X, its growth $EX = \varepsilon X \setminus X$ of the compactification εX is a Z-set in εX.

Theorem 2. The compactification εM of any Q-manifold M is the Hilbert cube of Q, i.e., $\varepsilon M \simeq Q$.

Corollary 3. The compactification $\varepsilon(Q \times [0,1))$ is homeomorphic to Q i.e., $\varepsilon(Q \times [0,1)) \simeq Q$.

If X is an ANR-space, then $X \times Q$-is a Q manifold [6], therefore.

Corollary 4. For any non-compact ANR-space X, the space $\varepsilon(X \times Q)$ is homeomorphic to the Hilbert cube Q i.e. $\varepsilon(X \times Q \simeq Q)$.

For a compact X, $P(X)$ denotes the space of all probability measures, $P_n(X) = \{\mu \in P(X) : |\operatorname{supp}\mu| \leq n\}, P_\omega(X) = \bigcup_{n=1}^{\infty} P_n(X), \operatorname{supp}(\mu)$-the support of the measure [3].

Theorem 3. For any infinite compact X we have:

a) the compactification $\varepsilon\big(P(X) \setminus P(A)\big)$ of $P(X) \setminus P(A)$ is homeomorphic to Q, where $A \subset X$, $A \neq X$, $\overline{A} = A$; i.e. $\varepsilon\big(P(X) \setminus P(A)\big) \simeq Q$;

b) $\varepsilon(S_P(A))$ is homeomorphic to Q, where $A \subset X$, $A \neq X$ A is open in X; i.e. $\varepsilon(S_P(A))$;

c) $\varepsilon\big(P(X) \setminus P_n(X)\big) \simeq Q$, where $n \in N$;

Theorem 4. For any infinite compact X, the compactification $\varepsilon(P_\omega(X))$ is homeomorphic to the Hilbert cube Q. i.e. $\varepsilon(P_\omega(X)) \simeq Q$.

If X is connected, locally compact and locally connected, then X is homotopy dense in εX [11].

Theorem 5. Let X be an infinite compact set and X_0 be a countable dense subset of it. i.e. $|X_0| = \chi_0, \quad X_0 \subset X$ and $\overline{X_0} = X$. Then

a) $\alpha(P_\omega(X_0)) \simeq Q$;

b) $\alpha(P(X_0)) \simeq Q$;

Theorem 6. Let X be an infinite metric compact set such that $P(X) \simeq Q$. Then

a) for any $n \in N$ the subspace $P_n(X)$ is a Z-set in $P(X)$:

b) For any proper subset A of the compact X the subspace $S_P(A)$ is homotopy dense in $P(X)$.

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Regularization of a multidimensional inverse problem with a hyperbolic-type operator

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Let the inverse problem of the form:

$$u_{tx^2y} + \lambda u_{x^2y} = \lambda_1 u_y + \Phi_0(u, u_x, u_{x^2}) + f(t)(H\theta)(x,y), \tag{1}$$

$$\begin{cases} u\,(0,y,t) = u_x\,(0,y,t) = 0, \\ u\,(x,0,t) = 0; u\,(x,y,0) = \varphi(x,y), \forall(x,y,t) \in \bar{\Omega}, (\Omega = (0,X) \times (0,b) \times (0,\infty)), \\ \varphi(0,y) = \varphi(x,0) = \varphi(0,0) = 0, (\varphi_x(0,y) = 0; u\,(t,0,0) = 0; u\,(0,0,0) = 0), \end{cases} \tag{2}$$

$$\begin{cases} (u_t + \lambda u)\left|_{t=T}\right. = g\,(x,y)\,, \forall(x,y) \in \bar{D}_1, \\ g\left|_{x=0}\right. = g\left|_{y=0}\right. = 0, (g(0,0) = 0; D_1 = (0,X) \times (0,b); T \in (0,\infty)), \end{cases} \tag{3}$$

with respect to the known functions $\lambda, \lambda_1, \Phi_0, f, \varphi, g, K$ the following conditions are allowed:

$$\begin{cases} C(D_0) \ni K(x,\tau,y,\nu) : |K(.)| \le C_{01}, \forall(x,\tau,y,\nu) \in D_0, \\ D_0 = \{(x,\tau) : 0 \le \tau \le x \le X\} \times [0,b] \times [0,b], \\ K(.) \ge 0; K(0,0,y,y) \ne 0, \forall y \in [0,b], \\ C^{1,1,1}(D_2) \ni \Phi_0(u_1,u_2,u_3), (D_2 = R \times R \times R), |\Phi_0| \le \beta_1, |\Phi_{0u_i}| \le \beta_2, (i = \overline{1,3}), \\ C^{2,1}(\bar{D}_1) \ni \varphi(x,y); \sup\limits_{\bar{D}_1}[|\varphi(.)|\,, |\varphi_x|\,, |\varphi_{x^2}|\,, |\varphi_y|\,, |\varphi_{xy}|\,, \left|\varphi_{x^2y}\right|] \le \beta_3, \\ C^{1,1}(\bar{D}_1) \ni g(x,y); \sup\limits_{\bar{D}_1}[|g(.)|\,, |g_x(.)|\,, |g_y(.)|\,, |g_{xy}(.)|] \le \beta_4, \\ C(R_+) \ni f(t) : 0 \le f(t) \le f_{01} < \infty : \int\limits_{R_+} f(t)dt \le f_{02} = const, \\ f(T) \ne 0; \|f(t)\|_C \left|f^{-1}(T)\right| \le \beta_5; f_0 = \max(f_{01}, f_{02}), \beta_0 = \max \beta_i, (i = \overline{1,5})). \end{cases} \tag{4}$$

The unknown is the vector function $P = (u, \theta)$ with two components. Since the inverse problem (1)-(3) degenerates into an ill-posed Volterra integral equation of the first kind with a non-negative solution, the regularizability of this integral equation is considered in the generalized sense in the space $Z^2(D_1)$.

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The use of educational research in teaching geometry and topology

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The problem of using educational research in teaching geometry and topology elements with the wide use of interactive computer programs is considered. Research work in a geometry lesson is one of the forms of setting and solving a problem, in the process of solving which a stimulating environment is created for the mental development of students, the disclosure of their personal qualities. When conducting educational research, priority is given to "open" problems, proofs of theorems, geometric constructions, as well as topological properties of figures. Such topological concepts as a geometric body and its surface, inner area, outer area, border are the basis for understanding and defining shapes in the school geometry course.

The use of educational research involves the wide use of a computer, various programs of dynamic geometry (interactive programs GeoGebra, "Live Mathematics", "Mathematical Constructor", etc.). As an example, consider the proof of the theorem on the sum of the internal angles of a triangle. Having built a geometric drawing in one of the dynamic geometry programs, students measure the angles of the resulting triangle and make sure that the sum will always be equal to 180^{o}, with any change in the configuration of the triangle. Then an analytical proof of the theorem is considered. In this case, the method of "computer proof" is used to conduct a computer experiment that establishes the fact that the sum of the angles of a triangle is independent of its parameters. By comparison, Chinese schools emphasize modeling and visualization, and the integration of modern technology into math teaching is a key element of learning effectiveness.

Thus, this work has implemented several didactic functions: teaching how to make and read geometric drawings; the formation of skills to notice properties and patterns, to deduce consequences from given conditions, to conduct evidential reasoning and draw conclusions.

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About uniformly Scheepers spaces

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A uniform space (X, U) is uniformly Scheepers if for each sequence $(\alpha_n | n \in N) \subset U$ there is a sequence $(\beta_n | n \in N)$ of finite sets such that for each n, $\beta_n \subset \alpha_n$ and the set $\{\bigcup \beta_n | n \in N\}$ is a ω-cover of X.

An open cover α of a space X is an ω-cover [1] if X does not belong to α and every finite subset of X is contained in an element of α.

Evidently that a uniform space (X, U) we have:
σ-precompact $\to$ uniformly Hurewicz $\to$ uniformly Scheepers $\to$
$\to$ uniformly Menger $\to$ pre-Lindelöf.

Theorem 1. For a uniform space (X, U) the following are equivalent:

1. (X, U) is uniformly Scheepers;
2. For each sequence $(\alpha_n | n \in N) \subset U$ there is a sequence $(\beta_n | n \in N)$ of finite sets such that for each n, $\beta_n \subset \alpha_n$ and the set $\{St(\bigcup \beta_n, \alpha_n) | n \in N\}$ is an ω-cover of X;
3. For each sequence $(\alpha_n | n \in N) \subset U$ there is a sequence $(F_n | n \in N)$ of finite

subsets of X such that $\{St(F_n, \alpha_n) | n \in N\}$ is an ω-cover of X.

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About Star-Menger Mappings

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Let $f : X \to Y$ be continuous mappings between topological spaces X and Y.

Definition 1. A mapping f is said to be a star-Menger mapping, if the mapping f is closed and preimage $f^{-1}y$ for each $y \in Y$ is star-Menger space.

Definition 2. A mapping f is said to be a weakly star-Menger mapping, if for each $y \in Y$ is star-Menger space.

It is clear that every star-Menger mapping is a weakly star-Menger mapping. Every strongly star-Menger mapping is star-Menger mapping.

If f is a star-Menger mapping and $Y = \{y\}$, then X is star-Menger space.

Lemma 1. For a mapping f the following are equivalent:

1. f is star-Menger mapping;

2. For each point $y \in Y$ and for each sequence $(\alpha_n | n \in N)$ of $f^{-1}y$ by of open sets in X there exists a sequence $(\beta_n | n \in N)$ and neighborhood O_y of the point y such that $\beta_n \subset \alpha_n$ for each $n \in N$ and $\bigcup\limits_{n \in N} St(\bigcup \beta_n, \alpha_n) = f^{-1}O_y$.

Theorem 1. Let$f : X \to Y$ be a mapping. If Y and f is star-Menger, then X is also have star-Menger property, conversely, if X is star-Menger space, then the mapping f is a star-Menger.

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About compact type uniformly continuous mappings

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u-uniformly compact, u-uniformly finally compact and u-uniformly countable compact spaces are studied in the work [3]. In this work we introduce and study u-uniformly compact, u-uniformly finally compact and u-uniformly countable compact mappings of the uniform spaces.

Let$f : (X, U) \to (Y, V)$ be an arbitrary uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V).

A uniformly continuous mapping f is called u-uniformly compact (u-uniformly finally compact) if for any finitely additive open cover α of the space (X, U) there exist finitely additive uniform cover β of the space (Y, V) and finite (countable) uniform cover γ of the space (X, U), such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

A uniformly continuous mapping f is called u-uniformly countable compact if for any finitely additive countable open cover α of the space X there exist finitely additive uniform cover β of the space Y and finite uniform cover γ of the space X, such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

Theorem 1. If f and (Y, V) are u-uniformly compact, then the space (X, U) is also u-uniformly compact, conversely, if (X, U) u-uniformly compact, then f is also u-uniformly compact.

Theorem 2. If f and (Y, V) are u-uniformly finally compact, then the space (X, U) is also u-uniformly finally compact, conversely, if (X, U) u-uniformly finally compact, then f is also u-uniformly finally compact.

Theorem 3. If f and (Y, V) are u-uniformly countable compact, then the space (X, U) is also u-uniformly countable compact, conversely, if (X, U) u-uniformly countable compact, then f is also u-uniformly countable compact.

Theorem 4. A uniformly continuous mapping f is u-uniformly compact if and only if it is u-uniformly finally compact and u-uniformly countable compact.

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About sequentially complete mappings

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This work investigates sequentially complete mappings of uniform spaces.

Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping. If F is Cauchy filter, then $fF = \{fN : N \in F\}$ is a Cauchy filter base.

A uniformly continuous mapping $f : (X, U) \to (Y, V)$ of a uniform space (X, U) to a uniform space (Y, V) is called sequentially complete if every Cauchy filter F in (X, U) for which fF converges in (Y, V), converges in (X, U).

Consider the following square in the category If $Unif$: Cauchy filter, then

$$\begin{array}{ccc} (X,U) & \overset{i_X}{\to} & (\hat{X},\hat{U}) \\ f\downarrow & & \downarrow\hat{f} \\ (Y,V) & \overset{i_Y}{\to} & (\hat{Y},\hat{V}) \end{array} \qquad (1)$$

where $(\hat{X}, \hat{U})$ and $(\hat{Y}, \hat{V})$ be the sequentially completions of uniform spaces (X, U) and (Y, V) respectively, i_X and i_Y are identical uniform embeddings of (X, U) and (Y, V) in $(\hat{X}, \hat{U})$ and $(\hat{Y}, \hat{V})$ respectively, and $\hat{f}$ is a unique extensions of the mapping f . The square (1) is commutative.

Theorem 1. For a uniformly continuous mapping $f : (X, U) \to (Y, V)$ of a uniform space (X, U) onto a uniform space (Y, V) the following are equivalent:
1. The mapping f is sequentially complete;
2. $\hat{f}(\hat{X}\backslash X) \subset \hat{Y}\backslash Y$;
3. The square is Cartesian in the Category $Unif$.

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z_U-filters

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In this work we studyz_U-filters.

Definition 1. Nonempty subsystem $F \subset Z_U$ called is z_U-filter of the uniform space (X, U), if the following are met:
1. $\varnothing \notin F$;
2. If $Z_1, Z_2 \in F$, then $Z_1 \bigcap Z_2 \in F$;
3. If (X, U) and $Z' \in Z_U$ such that $Z \subset Z'$, then $Z' \in F$.

Theorem 1. If I is ideal in $C_U(X)$, then the family $F_I = \{Z_f : f \in I\}$ is z_U-filter on the uniform space (X, U), conversely, if F is z_U-filter on the uniform space (X, U), then system $I_F = \{f \in C_U(X) : Z_f \in F\}$ is ideal in $C_U(X)$.

Theorem 2. If I is maximal ideal in $C_U(X)$, then the family $F_I = \{Z_f : f \in I\}$ is z_U-ultrafilter on the uniform space (X, U), conversely, if F is z_U-ultrafilter on the uniform space (X, U), then system $I_F = \{f \in C_U(X) : Z_f \in F\}$ is maximal ideal in $C_U(X)$.

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τ-boundedness of the mapping of hyperspace of uniform space

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In this work we study some classes of uniformly continuous mappings of hyperspaces of uniform spaces.

Let (X, U) be a uniform spaces and $\exp_c X$ - set of all compact spaces of the space (X, τ_U). For each $\alpha \in U$ we put $O(\alpha) = \{\langle\alpha'\rangle : \alpha' \subset \alpha - finite\}$, where $\langle\alpha'\rangle = \{K \in \exp_c X : K \subset \bigcup \alpha', K \subset A, A \in \alpha'\}$. If B is base of the uniform spaces (X, U), then $O(B) = \{O(\alpha) : \alpha \in B\}$ forms the base of some uniformity $\exp_c U$ on $\exp_c X$.

Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). The mapping f is called τ-bounded, if for each $\alpha \in U$ there exist a uniform cover $\beta \in V$ and a uniform cover $\gamma \in U$cardinality $\leq \tau$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$ [3].

Theorem 1. Let $f : (X, U) \to (Y, V)$ be a τ-bounded mapping of the uniform space (X, U) onto uniform space (Y, V). Then mapping $\exp_c(f) : (\exp_c X, \exp_c U) \to (\exp_c Y, \exp_c V)$ of the hyperspace onto hyperspace $(\exp_c Y, \exp_c V)$, where $\exp_c(f) = [K]$, $K \in \exp_c X$ is τ-bounded.

Corollary 1. Let $f : (X, U) \to (Y, V)$ be a precompact mapping of the uniform space (X, U) onto uniform space (Y, V). Then mapping $\exp_c(f) : (\exp_c X, \exp_c U) \to (\exp_c Y, \exp_c V)$ of the hyperspace onto hyperspace $(\exp_c Y, \exp_c V)$, where $\exp_c(f) = [K]$, $K \in \exp_c X$ is precompact.

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About remainders of the uniformly continuous mappings

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In this work we study remainders of uniformly continuous mappings.

A uniformly continuous mapping $f:(X,U)\to(Y,V)$ of uniform space (X,U) onto a uniform space (Y,V) is called precompact, if for each $\alpha\in U$ there exist a uniform cover $\beta\in V$ and finite uniform cover $\gamma\in U$, such that $f^{-1}\beta\wedge\gamma\succ\alpha$ [1]; a uniformly continuous mapping $f:(X,U)\to(Y,V)$ of uniform space (X,U) onto a uniform space (Y,V) is called uniformly perfect if it is precompact and perfect [1].

Throughout this paper all uniform spaces are assumed to be Hausdorff and mappings are uniformly continuous.

Let $f:(X,U)\to(Y,V)$ be a uniformly continuous mapping of a uniform space (X,U) onto a uniform space (Y,V). Let $\nu f:(\nu X,\nu U)\to(Y,V)$ be a Hewitt realcompactification of the mapping f. We denote by $\nu f|_{\nu X\setminus X}:(\nu X\setminus X,\nu U_{\nu X\setminus X})\to(Y,V)$ the remainder of the mapping f.

Theorem 1. The remainder $\nu f|_{\nu X\setminus X}:(\nu X\setminus X,\nu U_{\nu X\setminus X})\to(Y,V)$ of the uniformly continuous mapping $f:(X,U)\to(Y,V)$ is Hewitt complete if and only if the uniform space (X,U) is open subspace of the space $(\nu X,\nu U)$.

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Strongly μ-complete space

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In this paper, strongly μ-complete spaces are studied.

A filter F in a uniform space (X, U) is called a weak Cauchy filter if for any $\alpha \in U$ there exists $A \in \alpha$ such that $A \bigcap B \neq \varnothing$ for any$B \in F$.

A uniform space (X, U) is called a strongly complete space if any weak Cauchy filter converges in it.

A uniform space (X, U) is called sequentially strongly μ-complete if every weak Cauchy filter F in (X, U) having a base cardinality $\leq \mu$ converges.

It is clear that every strongly complete space is strongly μ-complete.

Theorem 1. A uniform space(X, U) is strongly μ-complete if and only if it is uniformly μ-paracompact i.e., every finitely additive countable open cover cardinality $\leq \mu$ is a uniform cover.

Corollary 1. A uniform space(X, U) is sequentially strongly complete if and only if it is sequentially uniformly paracompact.

Theorem 2. A Tychonoff space X is μ-paracompact if and only if a uniform space (X, U_X) with universal uniformity U_X is strongly μ-complete.

Corollary 2. A Tychonoff space X is sequentially paracompact if and only if a uniform space (X, U_X) with universal uniformity U_X is sequentially strongly complete.

Theorem 3. Any uniformly μ-paracompact space is μ-complete.

Corollary 3. Any countable uniformly paracompact space is sequentially μ-complete.

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On a Uniform Analogue of Strongly Paracompact Spaces

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In this work we introduce and study strongly uniformly $\tau - P$-finally paracompact space. This concept generalizes the concept of strongly uniformly P-paracompact spaces.

A uniform space (X, U) is said to be a strongly uniformly P-paracompact space if it is uniformly P-paracompact and the topological space (X, τ) is strongly paracompact.

Definition 1. A uniform space (X, U) is said to be a strongly uniformly $\tau - P$-finally paracompact space if it is uniformly $\tau - P$-finally paracompact and the topological space (X, τ) is strongly τ-finally paracompact.

A topological space (X, τ) is said to be a strongly τ-finally paracompact if every open cover has a star finite open refinement cardinality $\leq \tau$.

Every strongly uniformly $\tau - P$-finally paracompact space is uniformly $\tau - P$-finally paracompact.

Proposition 1. Every strongly uniformly $\tau - P$-finally paracompact space is strongly uniformly $\tau - B$-finally paracompact.

Theorem 1. The uniform space (X, U) is strongly uniformly $\tau - P$-finally paracompact if and only if for each open cover ω of space (X, U) there exists a uniform continuous ω-mapping f of the uniform space (X, U) onto some strongly τ-finally paracompact metrizable uniform space (X, U).

Theorem 2. The preimage of a strongly uniformly $\tau - P$-finally paracompact space under perfect mappings is strongly uniformly $\tau - B$-finally paracompact.

Theorem 3. The image of a strongly uniformly $\tau - P$-finally paracompact space under a strongly uniformly open uniformly perfect mappings is strongly uniformly $\tau - P$-finally paracompact.

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Dimensions of uniformly continuous mappings

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In this work dimensions of uniformly continuous mappings are introduced and studied.

A uniform space (X, U) has a uniformly dimension $\leq n$ ore $\dim U \leq n$ if the uniformity U has base B consisting of coverings α such that $ord\alpha \leq n+1$ where $ord\alpha = |\{A \in \alpha : A \ni x\}|$ for each $x \in X$.

Let $f : (X, U) \to (Y, V)$ be an arbitrary uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). A pseudouniformity $U_f \subset U$ is called a base of a uniformly continuous mapping f if for any $\alpha \in U$ there exist $\beta \in V$ and $\gamma \in U_f$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

Definition 1. A uniformly continuous mapping $f : (X, U) \to (Y, V)$ a uniform space (X, U) onto a uniform space (Y, V) is called $\dim f \leq n$ if the mapping f has base U_f consisting of coverings α such that $ord\alpha \leq n+1$.

Let $f : (X, U) \to (Y, V)$ be an arbitrary uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). If $\dim f \leq n$ and $Y = \{y\}$ then $\dim U \leq n$. Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V) and$g : (Y, V) \to (Z, W)$ be a uniformly continuous mapping of a uniform space (Y, V) onto a uniform space (Z, W). If $\dim f \leq n$ and $\dim g \leq n$ then $\dim g \circ f \leq n$.

Theorem 1. Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). If $\dim f \leq n$ and $\dim V \leq n$, then $\dim U \leq n$.

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About τ-star mappings

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Throughout this work all uniform spaces are assumed to be Hausdorff and mappings are uniformly continuous.

A uniform cover α of the uniform space (X, U) is called τ-star uniform cover if $|St(\alpha, x)| \leq \tau$ for each $x \in X$, where $St(\alpha, x) = \{A \in \alpha : A \ni x\}$, a uniform cover α of the uniform space (X, U) is called strongly τ-star uniform cover if $|St(\alpha, A)| \leq \tau$ for each $A \in \alpha$, where $St(\alpha, A) = \{A' \in \alpha : A \bigcap A' \neq \varnothing\}$.

A uniform space (X, U) is called (strongly) τ-star uniform space if U have a base B consisting of (strongly) τ-star uniform cover.

Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). The mapping f is called precompact, if for each $\alpha \in U$ there exist a uniform cover $\beta \in V$ and a finite uniform cover $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$; uniformly perfect, if it is both precompact and perfect.

Definition 1. The mapping f is called τ-star uniformly mapping, if for each $\alpha \in U$there exist a uniform cover $\beta \in V$ and τ-star uniform cover $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

The mapping f is called strongly τ-star uniformly mapping, if for each $\alpha \in U$ there exist a uniform cover $\beta \in V$ and strongly τ-star uniform cover $\gamma \in U$ such that $f^{-1}\beta \wedge \gamma \succ \alpha$.

Every uniformly prefect mapping is strongly τ-star uniformly and every strongly τ-star uniformly mapping is τ-star uniformly.

Theorem 1. Let $f : (X, U) \to (Y, V)$ be a uniformly continuous mapping of a uniform space (X, U) onto a uniform space (Y, V). If f and (Y, V) is τ-star uniformly, then the space (X, U) is also a τ-star uniformly, conversely, if the space (X, U) is τ-star uniformly then mapping f also is τ-star uniformly.

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